Distributed Algorithms

1) To prove that the given **k-set consensus algorithm** is **f-resilient to crash failures**, we must demonstrate two key properties:

1. **Validity**: Every output ​ is one of the initial inputs
2. **k-Agreement**: The set of all outputs contains at most kkk distinct values.

At each round, processors merge their V sets by taking the union of their own V with the sets received from other processors.

At the final round (r), each processor outputs y=min(V).

**Validity** is easy to show because all he output is the min v and because we assume only faulty nodes the values must have been recive by an input.

**k-Agreement** is harder to prove, but it is close to what we saw during the lecture. Because there are rounds, there is at least one round where there are fewer than k faulty nodes. Because of this, the maximum number of different values can be k at this round. The reason for this is that all the good nodes will have values between one and k values, either receiving at most kk−1 values in the worst case, with the other nodes adding one more value.

1. **The Critical Round with Fewer Faulty Nodes:**
   * Since the algorithm runs rounds, there must be at least **one round where fewer than k faulty nodes influence the communication**. This is because the failures are spread across f faulty nodes, and the rounds allow enough time for information from non-faulty nodes to propagate.
2. **Bounding the Number of Values:**
   * In the worst case, a set V at any node can include at most k distinct values during this critical round. Here's why:
     + All non-faulty nodes collectively merge their V sets by receiving values from each other, and the number of distinct values in V cannot increase beyond the initial contributions from non-faulty nodes and the influence of k−1 faulty nodes.
     + Even if the k−1 faulty nodes attempt to introduce new values, the propagation ensures that no more than k distinct values can exist across all V sets by the end of the critical round.

2)

a) To show that no consensus algorithm satisfies non-faulty-node validity, we just need to demonstrate that the trust between nodes creates an issue. Assume a function where the nodes must agree on an output. The Byzantine node can send an output that is not in x (the set of non-faulty node inputs), which disrupts the validity condition.

b) To prove this, we need to show that for a given algorithm A, in all cases, non-faulty validity equals all-same validity.

We will analyze each case, assuming nnn non-faulty nodes:

1. If all nodes have the same value (0 or 1), non-faulty validity means the output must match the input of one of the nodes. This implies the output will be either 0 or 1 accordingly (if all nodes have 0, the output is 0; if all nodes have 1, the output is 1). Similarly, all-same validity means that if all nodes have the same value, the output is the same as their value (e.g., if all have 0, the output is 0, and vice versa).
2. Assume kkk nodes have the value 0 and n− k nodes have the value 1, where 0<k<n. Non-faulty validity means the output must be either 0 or 1 (matching one of the non-faulty inputs). However, all-same validity has no restriction on the output because not all non-faulty nodes have the same value.

This shows the equivalence of non-faulty validity and all-same validity in cases where all nodes agree, and the divergence when they do not.

c) Improvments:  **Increase Wireless Transmission Bandwidth**

* **Rationale**: If the N1 implant could transmit data at higher rates (e.g., 5 Mbps instead of 1 Mbps), the compression challenge would become significantly less complex. Increasing the transmission bandwidth would allow for less aggressive compression, reducing potential data loss and computational demands.
* **Implementation**: Explore advancements in wireless communication technologies, such as 5G or ultra-wideband (UWB) systems, that are both high-speed and low-power. Additionally, optimize antenna designs and signal processing techniques to improve throughput without compromising the implant's energy efficiency.

 **Research in Information Theory**

* **Rationale**: Information theory provides a theoretical foundation for understanding data redundancy, entropy, and compression limits. By applying advanced principles like Shannon's entropy or modern research in channel coding, Neuralink can develop compression algorithms tailored to the neural signal domain.
* **Implementation**: Invest in interdisciplinary research combining neuroscience, information theory, and signal processing. Collaborate with academic institutions or hire domain experts to identify innovative methods that maximize data compression while preserving neural signal integrity.